Causal Theories of Actions Revisited

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Abstract

It has been argued that causal rules are necessary for representing both implicit side-effects of actions and action qualifications, and there have been a number different approaches for representing causal rules in the area of formal theories of actions. These different approaches in general agree on rules without cycles. However, they differ on causal rules with mutual cyclic dependencies, both in terms of how these rules are supposed to be represented and their semantics. In this paper we show that by adding one more minimization to Lin's circumscriptive causal theory in the situation calculus, we can have a uniform representation of causal rules including those with cyclic dependencies. We also demonstrate that sometimes causal rules can be compiled into logically equivalent (under a proposed semantics) successor state axioms even in the presence of cyclical dependencies between fluents.

Introduction

Reiter (2001) argues that to solve many reasoning problems about actions, it is convenient to work with the precondition axioms and the successor state axioms. For each *action* function $A(\vec{x})$, a precondition axiom (PA) has a syntactic form

$$Poss(A(\vec{x}), s) \equiv \Pi_A(\vec{x}, s).$$

(An *action* $A(\vec{x})$ is possible in situation s if and only if $\Pi_A(\vec{x}, s)$ holds in s, where $\Pi_A(\vec{x}, s)$ is a formula with free variables among $\vec{x}$ and s.[1]) *Situations* are first order (FO) terms which denote possible world histories. A distinguished constant S_0 is used to denote the *initial situation*, and function $do(a, s)$ denotes the situation that results from performing action a in situation s. Every situation corresponds uniquely to a sequence of actions. Moreover, notation $s' \preceq s$ means that either situation s' is a subsequence of situation s or $s = s'$. There are axioms Σ for situations which characterize situations as a single finitely branching infinite tree starting from S_0 such that at each node S, each branch corresponds to new situation $do(A, S)$ arising from execution of A, one of finitely many actions, at S (Reiter 2001). These *foundational* axioms for situations are domain independent. *Objects* are FO terms other than actions and situations that depend on the domain of application. Above, $\Pi_A(\vec{x}, s)$ is a formula *uniform* in situation argument s: it does not mention the predicates $Poss$, $\prec$ or $Caused$ (introduced below), it does not quantify over variables of sort situation, it does not mention equality on situations, and it has no occurrences of situation terms other than the variable s (see (Reiter 2001)). We also call a formula, that is uniform in situation argument s, a *state* formula, interchangeably. For each fluent $F(\vec{x}, s)$, a successor state axiom (SSA) has a syntactic form

$$F(\vec{x}, do(a, s)) \equiv [\exists \vec{y}_i](a = PosAct_i(\vec{t}_i) \wedge \phi_i^+(\vec{x}, \vec{y}_i, s)) \vee$$
$$F(\vec{x}, s) \wedge \neg[\exists \vec{z}_j](a = NegAct_j(\vec{t'}_j) \wedge \phi_j^-(\vec{x}, \vec{z}_j, s)),$$

where each $\phi_i^+(\vec{x}, \vec{y}_i, s)$ ($\phi_j^-(\vec{x}, \vec{z}_j, s)$, respectively) is a formula uniform in s, and each $PosAct(\vec{t}_i)$ ($NegAct(\vec{t'}_j)$, respectively) is an action term that makes $F(\vec{x}, do(a, s))$ true (false, respectively) if the context condition $\phi_i^+(\vec{x}, \vec{y}_i, s)$ ($\phi_j^-(\vec{x}, \vec{z}_j, s)$, respectively) is satisfied. In this axiom, each $\vec{t}_i$ ($\vec{t'}_j$, respectively) is vector of terms including variables among $\vec{x}$ and quantified new variables $\vec{y}_i$ ($\vec{z}_j$, respectively), if there are any. In a general case, there might be at most a finite number of positive or negative effects on each fluent. In addition to Σ, PAs and SSAs, Reiter (2001) includes also into his Basic Action Theories (BATs), a finite set of FO formulas whose only situation term is S_0 (*initial* theory). This set of formulas specifies the values of all fluents in the initial state. It also describes all the facts that are not changeable by any actions in the domain. Finally, BATs include *unique name axioms* (UNA) for actions specifying that two actions are different if their names are different, and identical actions have identical arguments.

It has been observed that sometimes, an axiomatizer has to start not with PAs and SSAs, but with a different set of axioms representing (domain) state constraints (Finger 1986). For example, (McIlraith 2000) argues that it is more convenient for an axiomatizer to start with state constraints that characterize a complex technical or software system. She also demonstrates when a syntactically restricted set of situation calculus constraints and effect axioms can be compiled into a set of SSAs. From another perspective, (Baader et al. 2005b; 2005a) investigate how reasoning about actions can be carried out in description logics. The authors embed reasoning problems from the general situation calculus into a description logic setting. For the sake of simplicity, they consider only a special case of domain constraints that correspond to a set of acyclic definitions between concept-like fluents. In description logics, this set of axioms is called an *acyclic TBox*. Axioms in a TBox express general knowl-

[1] Here and subsequently, all free variables (typically written in lower case letters) including *object* variable $\vec{x}$, *situation* variable s, and a variable of sort *action* a are implicitly $\forall$-quantified at front of formulas.

edge about a domain and may include both terminological definitions and constraints that should hold after execution of arbitrary actions. A recent paper (Baader, Lippmann, and Liu 2010) proposes a generalization to a TBox that consists of *general concept inclusion* (GCI) axioms. It is important to observe that in description logics, it is the set of state constraints in TBox that is a primary concern of an axiomatizer.

State (or domain) constraints are traditionally facts that are true in every possible state. In the situation calculus, they are normally represented as first-order sentences with universal quantifiers over situations. For instance, the fact that no object x can be at two different locations l, l' in the same situation s can be represented as:

$$at(x, l, s) \wedge at(x, l', s) \supset l = l'. \tag{1}$$

However, surprisingly, not all state constraints are created equal. (Ginsberg and Smith 1988) (see also (Lin and Reiter 1994)) first point out that while some of them contribute to indirect effects of actions (called *ramification state constraints* in (Lin and Reiter 1994)), others serve as implicit qualifications on actions (called *qualification state constraints* in (Lin and Reiter 1994)). For instance, consider the action $move(x,l)$ that moves the object x to the location l

$$Poss(move(x, l), s) \supset at(x, l, do(move(x, l), s)).$$

(If the action $move(x, l)$ is possible (executable), then after it is performed the object x will be at location l.) Then the above state constraint about uniqueness of a location is a ramification one, for it should be used to imply the following indirect effect of $move(x, l)$:

$$\begin{array}{l} Poss(move(x, l), s) \supset \\ \quad l' \neq l \supset \neg at(x, l', do(move(x, l), s)). \end{array}$$

Now suppose that each location can have just one object:

$$at(x, l, s) \wedge at(y, l, s) \supset x = y. \tag{2}$$

Then this constraint about uniqueness of an object occupying l should be a qualification one, for it should be used to derive the following qualification on the action:

$$Poss(move(x, l), s) \supset \neg(\exists y) y \neq x \wedge at(y, l, s).$$

(One cannot move an object to a location which is already occupied by another object, for otherwise, the last state constraint will be violated.)

What is disturbing here is that although our intuitions about how the two state constraints should be used are different, they are represented in the same way. It seems clear that these two kind of state constraints are fundamentally different, and should be represented in fundamentally different ways.

Moreover, several researchers argued that the indirect effects of actions should be represented differently (e.g. (Baral 1995; Lin 1995; McCain and Turner 1995; Sandewall 1994; Thielscher 1995)). In particular, Lin (1995) argued that the indirect effects of actions cannot be faithfully described using ramification state constraints alone, and proposed to use *causal rules* to specify the constraints. The method proposed in (Lin 1995) is illustrated with several examples of how causal rules together with direct effect axioms can be successfully compiled into PAs and SSAs, and later implemented (Lin 2003). Once this compilation step has been completed, the resulting BAT can be subsequently used for reasoning about actions. However, the general results about applicability of this approach are stated only for acyclic (stratified) sets of causal rules. As soon as there are fluents with mutual causal dependencies, the approach proposed in (Lin 1995) is no longer applicable. The goal of this paper is to elaborate the approach proposed in (Lin 1995), so that an arbitrary finite set of causal rules can be handled as well. Ultimately, we are looking for computational mechanisms that can take an arbitrary finite set of causal rules and a finite set of direct effect axioms on the input and can compile them into a set of PAs and a set of SSAs. This paper can be considered a first step in this direction. Subsequently, we concentrate on solving the ramification problem only, and do not consider explicit action qualification axioms.

The paper is organized as follows. Section 2 discusses our approach in more details. In Section 3, we illustrate our approach on several simple examples. In Section 4, we consider a special syntactic case of causal rules and show that under a stated syntactic restriction, the causal rules can be compiled into SSAs, even if there are cyclic dependencies. Section 5 includes discussion and comparison with previously proposed solutions to the ramification problem.

The Method

In this section, it is convenient for us to consider a sort *fluent* in addition to sorts *action, object, situation*. Following (McCarthy and Hayes 1969), we also use the binary predicate $Holds(f, s)$ to say that a fluent f holds in s. Notice that in the introduction, we wrote, for instance, $at(x, l, s)$ instead of $Holds(at(x, l), s)$. We consider the former to be a shorthand for the latter. We shall continue to do so in an effort to improve the readability of our formulas. Formally, if F is a fluent name of arity $object^n \rightarrow fluent$, then we define the expression $F(t_1, ..., t_n, s)$ to be a shorthand for the formula $Holds(F(t_1, ..., t_n), s)$, where $t_1, \cdots, t_n$ are terms of sort $object$, and s is a term of sort $situation$.

We consider causal theories of the following form:

- Σ, the set of foundational axioms.
- A set of direct action effect axioms of the form:

  $$\Phi(s) \supset Caused(F(\vec{x}), v, do(A(\vec{y}), s)), \tag{3}$$

  where Φ is a formula uniform in s, $F(\vec{x})$ is a fluent, $A(\vec{y})$ is an action, and v is a variable of sort *truth value*. Compared to (Lin 1995), we omit the predicate $Poss(A(\vec{y}), s)$ as a precondition of a direct effect, following (Reiter 2001) and (Lin 2008).
- Causal rules of the form:

  $$\Phi(s) \supset Caused(F(\vec{x}), v, s), \tag{4}$$

  where $\Phi(s)$ is a formula uniform in s, and F a fluent. Compared to (Lin 1995), we do not allow the predicate $Caused$ in the premises, but allow only arbitrary state formulas. We believe this simplifies the task of a knowledge engineer who is responsible for writing causal rules.

If both arbitrary state formulas and causation statements would be allowed in premises of causal rules, but there is no recipe which of them should be used when, then this permissiveness could create uncertainty for a knowledge engineer.

As in (Lin 1995), there are general axioms about $Caused$:

$$\mathcal{T} \neq \mathcal{F} \wedge \forall v. v = \mathcal{T} \vee v = \mathcal{F}, \quad (5)$$

$$Caused(f, \mathcal{T}, s) \supset Holds(f, s), \quad (6)$$

$$Caused(f, \mathcal{F}, s) \supset \neg Holds(f, s), \quad (7)$$

where (5) is the domain closure axiom for sort *truth-value*, and $\mathcal{T}$ and $\mathcal{F}$ are two constants of sort *truth-value*.

For each fluent $F(\vec{x})$, the generic frame axiom, called *pseudo-successor state* axiom, is

$$Holds(F(\vec{x}), do(a, s)) \equiv Caused(F(\vec{x}), \mathcal{T}, do(a, s)) \vee Holds(F(\vec{x}), s) \wedge \neg Caused(F(\vec{x}), \mathcal{F}, do(a, s)). \quad (8)$$

From this axiom, we see that to get a real SSA as in (Reiter 2001) for each fluent, we need to derive some definitions of $Caused(F(\vec{x}), \mathcal{T}, do(a, s))$ and $Caused(F(\vec{x}), \mathcal{F}, do(a, s))$ in terms of two state formulas on s, respectively. To achieve this, Lin (1995) proposed to circumscribe the $Caused$ predicate in a theory consisting of the above direct effect axioms (3) and causal rules (4), but not the pseudo-successor state (8) and general axioms (5)-(7). While this approach works for acyclic causal rules such as those in the suitcase example from (Lin 1995), it does not work when there are cycles as we will see from the examples in the next section.

We propose here to add a second minimization, and show that this solves the problem of cyclic causal rules. To present our approach, we first make precise Lin's approach.

Given a set T_0 of the direct effect axioms and causal rules of the forms (3) and (4), respectively, Lin's causal theory, written $\mathcal{C}_l(T_0)$ below, consists of foundational axioms Σ, the general axioms (5) - (7) about $Caused$, the pseudo-successor state axioms (8), and $CIRC(T_0, Caused)$, the circumscription of $Caused$ in T_0 with all other predicates fixed. (See (McCarthy 1986; Lifschitz 1985; 1994; Doherty, Łukaszewicz, and Szałas 1997) for details about circumscription.)

Since the formulas (3) and (4) in T_0 are Horn in the $Caused$ predicate, $CIRC(T_0; Caused)$ can be computed by a simple Clark predicate completion (Clark 1978; Reiter 1982) to yield the following formulas, two for each fluent F:

$$Caused(F(\vec{x}), v, S_0) \equiv \Phi_0(S_0), \quad (9)$$

$$Caused(F(\vec{x}), v, do(a, s)) \equiv \Phi_1(do(a, s)), \quad (10)$$

where Φ_0 and Φ_1 are computed as follows. Let the following be the list of direct effect axioms about F:

$$\phi_1(s) \supset Caused(F(\vec{x}), v, do(A_1(\vec{y_1}), s)),$$
$$\ldots$$
$$\phi_k(s) \supset Caused(F(\vec{x}), v, do(A_k(\vec{y_k}), s))$$

and the following the list of causal rules about F:

$$\psi_1(s) \supset Caused(F(\vec{x}), v, s),$$
$$\ldots$$
$$\psi_m(s) \supset Caused(F(\vec{x}), v, s).$$

Then $\Phi_0(S_0)$ is

$$\psi_1(S_0) \vee \cdots \vee \psi_m(S_0),$$

and $\Phi_1(do(a, s))$ is

$$[\phi_1(s) \wedge a = A_1(\vec{y_1})] \vee \cdots \vee [\phi_k(s) \wedge a = A_k(\vec{y_k})] \vee \psi_1(do(a, s)) \vee \cdots \vee \psi_m(do(a, s)).$$

Notice that if $m = 0$ (meaning no causal rules about F), then Φ_0 is $\perp$, where $\perp$ stands for $L \wedge \neg L$. If both $m = 0$ and $k = 0$, then Φ_1 is $\perp$.

In the following, given a set T_0 of direct effect axioms and causal rules of the forms (3) and (4), respectively, we denote by T_1 the set of equivalences (9) and (10).

We can now state our method as below:

1. Let T_1' be the result of replacing each atom of the form $Holds(F(\vec{t}), do(a, s))$ in T_1 by the right hand side of (8), i.e., with
$Caused(F(\vec{t}), \mathcal{T}, do(a, s)) \vee$
$Holds(F(\vec{t}), s) \wedge \neg Caused(F(\vec{t}), \mathcal{F}, do(a, s))$.
2. Our second minimization is then to circumscribe $Caused$ in T_1' with all the other predicates fixed, $CIRC(T_1', Caused)$.
3. Our final causal action theory $\mathcal{CAT}\ (T_0)$ will then consist of foundational axioms Σ, the general axioms (5) - (7) about $Caused$, the pseudo-successor state axioms (8), and $CIRC(T_1', Caused)$.

The following result says that our new causal theory is stronger than the one in (Lin 1995).

Theorem 1 $\mathcal{CAT}\ (T_0) \models \mathcal{C}_l(T_0)$

Proof: This follows from the following entailments:

$$CIRC(T_1'; Caused) \models T_1',$$
$$\{(8) \mid F \text{ is a fluent}\} \models T_1 \equiv T_1',$$
$$\models CIRC(T_0; Caused) \equiv T_1.$$

■

Thus if the method of (Lin 1995) yields a successor state axiom for each fluent, as when there are no cycles in causal rules, so will our new method. In this sense, our new approach indeed extends the one in (Lin 1995).

Examples

In this section, we would like to consider a few examples explaining our proposal. First of all, as mentioned above, for the suitcase example from (Lin 1995), our method yields exactly the same SSAs as in (Lin 1995).

Similarly, one can verify that for the complex electric circuit[2] from Figure 2.2 in (Thielscher 2000), our method also yields a SSA for each fluent.

Subsequently, we concentrate on examples of $\mathcal{CAT}$ where our new approach can produce SSAs, but the method from (Lin 1995) is not strong enough to do that.

[2]This circuit consists of a battery connected to a separate switch sw_0 that controls n parallel sub-circuits. Each sub-circuit contains its own switch sw_i connected to a light bulb l_i. If sw_0 is not up, then there is no light in any of the bulbs no matter what are the positions of their switches, but if sw_0 is up, then the fluent l_i is true if and only if sw_i is up.

A Chain Reaction

A chain reaction is any self-sustaining physical or chemical process such that its by-products cause the process to continue (with or without acceleration). There are many examples, but one of the simplest is an example of a fire started in a large pile of matches. Once a match inside a pile has been lit, it causes other surrounding matches to burn, and so on. To reason about fire in a pile, let x vary over the whole piles of matches, and let $fire(x,s)$ be a fluent that can become true after executing an action $ignite(x)$, but if it is true, then it becomes false after doing $extinguish(x)$ action. For the purposes of this example, we do not quantify over individual matches. In this example, a theory T_0 includes two direct effect axioms

$\neg fire(x,s) \supset Caused(fire(x), \mathcal{T}, do(ignite(x), s))$,
$fire(x,s) \supset Caused(fire(x), \mathcal{F}, do(extinguish(x), s))$,

a single causal rule with a cycle (fluent depends on itself):

$fire(x,s) \supset Caused(fire(x), \mathcal{T}, s)$.

It is easy to see that in this case $CIRC(T_0; Caused)$ yields the following:

$$\begin{array}{l} Caused(fire(x), v, S_0) \equiv\ v = \mathcal{T} \wedge fire(x, S_0),\\ Caused(fire(x), v, do(a,s)) \equiv\\ \quad a = extinguish(x) \wedge v = \mathcal{F} \wedge fire(x,s) \vee\\ \quad a = ignite(x) \wedge v = \mathcal{T} \wedge \neg fire(x,s) \vee\\ \quad v = \mathcal{T} \wedge fire(x, do(a,s)). \end{array}$$

According with Step 2 of our method, we have to replace $fire(x, do(a,s))$ in the last formula with

$$\begin{array}{l} Caused(fire(x), \mathcal{T}, do(a,s)) \vee\\ fire(x,s) \wedge \neg Caused(fire(x), \mathcal{F}, do(a,s)). \end{array}$$

But this yields a theory T_1' with the predicate $Caused$ defined in terms of itself. Consequently, the single minimization $CIRC(T_0; Caused)$ is not strong enough to produce a SSA for the fluent $fire(x)$. However, the second minimization $CIRC(T_1'; Caused)$ yields the formulas

$$\begin{array}{l} Caused(fire(x), \mathcal{T}, do(a,s)) \equiv\\ \qquad a = ignite(x) \wedge \neg fire(x,s),\\ Caused(fire(x), \mathcal{F}, do(a,s)) \equiv\\ \qquad a = extinguish(x) \wedge fire(x,s). \end{array}$$

Using these definitions, we can easily obtain a SSA for the fluent $fire(x)$ from the pseudo-successor state axiom (8).

Two Gear Wheels

In this well-known example by Denecker *et al.* (Belleghem, Denecker, and Dupré 1998), there are two interlocked gear wheels. We characterize each with a fluent $gw(n)$ meaning that the n-th gear wheel is turning. There are actions to initiate/halt rotation of wheels: $turn(n)$ and $block(n)$, respectively.

$Caused(gw(n), \mathcal{T}, do(turn(n), s))$,
$Caused(gw(n), \mathcal{F}, do(block(n), s))$,

Since the gear wheels are interlocked, rotation of one of the gear wheels causes another one to rotate too, but if one of them halts, the second one must halt too.

$gw(1,s) \supset Caused(gw(2), \mathcal{T}, s)$,
$gw(2,s) \supset Caused(gw(1), \mathcal{T}, s)$,
$\neg gw(1,s) \supset Caused(gw(2), \mathcal{F}, s)$,
$\neg gw(2,s) \supset Caused(gw(1), \mathcal{F}, s)$.

Let T_0 be a conjunction of these six axioms. Then, skipping axioms (9) related to S_0, $CIRC(T_0; Caused)$ yields

$$\begin{array}{l} Caused(gw(1), v, do(a,s)) \equiv\\ \quad a = turn(1) \wedge v = \mathcal{T}\ \vee\ gw(2, do(a,s)) \wedge v = \mathcal{T} \vee\\ \quad a = block(1) \wedge v = \mathcal{F}\ \vee\ \neg gw(2, do(a,s)) \wedge v = \mathcal{F}\ ,\\ Caused(gw(2), v, do(a,s)) \equiv\\ \quad a = turn(2) \wedge v = \mathcal{T}\ \vee\ gw(1, do(a,s)) \wedge v = \mathcal{T} \vee\\ \quad a = block(2) \wedge v = \mathcal{F}\ \vee\ \neg gw(1, do(a,s)) \wedge v = \mathcal{F}\ . \end{array}$$

As in the previous example, we observe that the first minimization does not allow us to compile direct effects and causal rules into SSAs. In Step 2, we replace $gw(1, do(a,s))$ and $gw(2, do(a,s))$ with the right hand sides of (8), and do some FO simplifications using general axioms about $Caused$. In the result, again skipping axioms (9) related to S_0, we get a theory T_1' that includes

$$\begin{array}{l} Caused(gw(1), v, do(a,s)) \equiv\\ \quad v = \mathcal{T} \wedge \big(a = turn(1) \vee a = turn(2) \vee\\ \qquad\qquad Caused(gw(1), \mathcal{T}, do(a,s)) \vee\\ \qquad\qquad gw(1,s) \wedge \neg Caused(gw(1), \mathcal{F}, do(a,s)) \vee\\ \qquad\qquad gw(2,s) \wedge \neg Caused(gw(2), \mathcal{F}, do(a,s)) \big) \vee\\ \quad v = \mathcal{F} \wedge \big(a = block(1) \vee a = block(2) \vee\\ \qquad\qquad Caused(gw(1), \mathcal{F}, do(a,s)) \vee\\ \qquad\qquad \neg gw(1,s) \wedge \neg Caused(gw(1), \mathcal{T}, do(a,s)) \vee\\ \qquad\qquad \neg gw(2,s) \wedge \neg Caused(gw(2), \mathcal{T}, do(a,s)) \big). \end{array}$$

and a similar axiom for $Caused(gw(2), v, do(a,s))$. In Step 3, we compute $CIRC(T_1'; Caused)$. This yields desirable definitions:

$$\begin{array}{l} Caused(gw(1), \mathcal{T}, do(a,s)) \equiv\\ \quad a = turn(1) \vee a = turn(2) \vee\\ \quad (gw(1,s) \vee gw(2,s)) \wedge \neg(a = block(1) \vee a = block(2)),\\ Caused(gw(1), \mathcal{F}, do(a,s)) \equiv\\ \quad a = block(1) \vee a = block(2) \vee\\ \quad (\neg gw(1,s) \vee \neg gw(2,s)) \wedge \neg(a = turn(1) \vee a = turn(2)). \end{array}$$

Again, to save space, we omit a similar axiom for $Caused(gw(2), v, do(a,s))$. Thus, in the two gear wheels example, we also computed successfully Reiter's SSAs.

The Firing Squad Example

The firing squad example is discussed in details in (Pearl 1999; 2009) to illustrate *structural causal models*, and different types of reasoning, including evaluation of counterfactual scenarios. Using the situation calculus, this example is also formulated in (Hopkins and Pearl 2007), where a new type of causal model is proposed to overcome limited (propositional) expressiveness of structural causal models. In (Hopkins and Pearl 2007), the example is formulated using SSAs and PAs only, without ramification state constraints. However, Pearl (1999; 2009) mentiones that causal mechanisms (laws) should be stated using sentences similar to domain constraints. In this section, we would like to consider a complementary translation of the firing squad example into the situation calculus that includes also causal rules of the form (4).

In a firing squad, there are two rifleman R_1 and R_2 who are accurate, alert, law abiding, and prepared to execute a prisoner P. There is an exogenous action *order* representing a court order. As soon as it arrives, the captain C gives a signal (represented as the ground action $signal(C)$ in axioms), and then both riflemen shoot the prisoner simultaneously and accurately (represented as action $shoot(x,y)$, x shoots y). We introduce the following fluents: $signaling(x,s)$ becomes true after doing $signal(x)$, both $shooting(x,y,s)$ and $dead(y,s)$ are true in the situation resulting from doing $shoot(x,y)$. In the initial theory, people are neither shooting, nor signaling and no one is dead: $\neg\exists y(dead(y,S_0))$, $\neg\exists x(signaling(x,S_0))$, $\neg\exists x,y(shooting(x,y,S_0))$. The example can be translated using two direct effect axioms

$Caused(signaling(C),\mathcal{T},do(order,s))$,
$Caused(shooting(x,y),\mathcal{T},do(shoot(x,y),s))$,

and three cuasal rules

$signaling(C,s) \supset Caused(shooting(R_2,P),\mathcal{T},s)$,
$signaling(C,s) \supset Caused(shooting(R_1,P),\mathcal{T},s)$,
$shooting(x,y,s) \supset Caused(dead(y),\mathcal{T},s)$.

These rules represent a kind of autonomous mechanisms. Once an initiating exogenous action has been executed, its effects propagate through the linked, interacting mechanisms. Let T_0 be conjunction of these five axioms. Since T_0 is Horn in $Caused$, the theory $T_1 = CIRC(T_0, Caused)$ in Step 1 includes the axioms

$$Caused(signaling(x),v,do(a,s)) \equiv \\ v=\mathcal{T} \wedge x=C \wedge a=order$$
$$Caused(shooting(x,y),v,do(a,s)) \equiv \\ v=\mathcal{T} \wedge \big(a=shoot(x,y) \vee \\ signaling(C,do(a,s)) \wedge y=P \wedge (x=R_1 \vee x=R_2)\big)$$
$$Caused(dead(y),v,do(a,s)) \equiv \\ \exists x.shooting(x,y,do(a,s)) \wedge v=\mathcal{T}.$$

(Here and subsequently, we omit all axioms related to S_0.) We can then obtain SSA for all fluents:

$$signaling(x,do(a,s)) \equiv\ x=C \wedge a=order \vee \\ signaling(x,s),$$
$$shooting(x,y,do(a,s)) \equiv\ a=shoot(x,y) \vee \\ y=P \wedge (x=R_1 \vee x=R_2) \wedge a=order \vee \\ signaling(C,s) \wedge y=P \wedge (x=R_1 \vee x=R_2) \vee \\ shooting(x,y,s),$$
$$dead(y,do(a,s)) \equiv\ \exists x(a=shoot(x,y)) \vee \\ y=P \wedge \big(a=order\ \vee signaling(C,s)\big) \vee \\ dead(y,s).$$

Now, as a variation of the firing squad example, suppose that whenever one rifleman is shooting, another is shooting as well:

$shooting(R_1,P,s) \supset Caused(shooting(R_2,P),\mathcal{T},s)$,
$shooting(R_2,P,s) \supset Caused(shooting(R_1,P),\mathcal{T},s)$.

In this case, the first minimization will not be strong enough to obtain intuitively correct SSAs, but the second minimization can handle the new cyclic rules without difficulties (similar to the gear wheels example). It should be easy to see that the second circumscritpion yields similar SSAs.

Following Pearl (1999; 2009), we can show that the $\mathcal{CAT}$ resulting from our translation of the firing squad example has reasonable logical consequences.

- Prediction (positive): If R_1 shot, then the prisoner is dead. Formally, $\mathcal{CAT} \models \forall s.dead(P,do(shoot(R_1,P),s))$
- Prediction (negative): If R_1 did not shot, then the prisoner is alive. Formally, $\mathcal{CAT} \models \forall s.\neg shooting(R_1,P,s) \supset \neg dead(P,s)$.
- Abduction: If the prisoner is alive, then the captain did not signal. Formally, $\mathcal{CAT} \models \forall s.\neg dead(P,s) \supset \neg signaling(C,s)$
- Transduction: If the rifleman R_1 shot, then the rifleman R_2 shot as well. Formally, $\mathcal{CAT} \models \forall s.(shooting(R_1,P,s) \supset shooting(R_2,P,s))$.
- Deliberate Action: If the captain gave no signal, but the rifleman R_1 still decides to shoot, then the prisoner will die and the rifleman R_2 will not shoot.
$$\mathcal{CAT} \models \forall s.\neg signaling(C,s) \supset \\ \big(\neg shooting(R_2,P,s) \wedge dead(P,do(shoot(R_1,P),s))\big).$$
- Counterfactual: If the prisoner is dead, then the prisoner would still be dead, even if the rifleman R_1 had not shot. Formally, $\mathcal{CAT} \models \forall s_a.\, dead(P,s_a) \supset \exists s_p.\, s_p \preceq s_a \wedge \forall s_h.s_h \neq s_a \supset \big(s_p \preceq s_h \wedge \neg\exists s' do(shoot(R_1,P),s') \preceq s_h \supset dead(P,s_h)\big)$.

  Notice that we can formulate in the situation calculus queries about counerfactual histories by using the precedence relation from the foundational axioms of Reiter (2001). Let s_a be an actual branch of the situation tree where P is dead, then there exists a past situation s_p (presumably, court order arrival, or the captain signaling) such that for all hypothetical situations s_h in the future of s_p if the sequence of actions leading to s_h does not include $shoot(R_1,P)$, then the prisoner still would be dead in s_h.

Related Work

Much work has been done on incorporating causal rules into action theories. Virtually every major action formalism has been extended with causal rules. Some of these previous works are discussed in details in (Giordano and Schwind 2004). Since our focus in this paper is on cyclic causal rules, we'll just consider the work that can deal with cyclic causal rules.

Recall that under our proposal, a causal rule has the form

$$\Phi(s) \supset Caused(F(\vec{x}),v,s),$$

where $\Phi(s)$ is a state formula uniform in s, meaning that it mentions only $Holds(f,s)$ but not the $Caused$ predicate. Thus, the causal rule that *open* is caused to be true when the two switches are up in the suitcase example is represented as:

$$up_1(s) \wedge up_2(s) \supset Caused(open,\mathcal{T},s),$$

and that the two gears are interlocked and one turning causes the other to turn is represented as

$gw(1,s) \supset Caused(gw(2),\mathcal{T},s)$,
$gw(2,s) \supset Caused(gw(1),\mathcal{T},s)$,
$\neg gw(1,s) \supset Caused(gw(2),\mathcal{F},s)$,
$\neg gw(2,s) \supset Caused(gw(1),\mathcal{F},s)$.

Notice that these formulas all have the same form. Intuitively, the one in the suitcase example is acyclic because there is no rule connecting *open* to up_1 or up_2. The ones in the gears example are cyclic. Formally, we can treat a set of causal rules of the form

$$l_1(s) \wedge \cdots \wedge l_n(s) \supset Caused(p,v,s) \qquad (11)$$

as a logic program, where p is a fluent atom, and l_i's are fluent literals, and define its dependency graph and loops, similar to what Lee did for McCain and Turner's causal theories (Lee 2004).

In McCain and Turner's causal logic (1997), the rule from the suitcase example would be represented as

$$up_1 \wedge up_2 \Rightarrow open,$$

and rules from the two gears example as

$$\top \Rightarrow gw(1) \equiv gw(2), \tag{12}$$

where $\top$ stands for $(L \vee \neg L)$ and represents propositional tautology. Compared to our representation, we see that cyclic and acyclic causal rules are represented differently in McCain and Turner's formalism. The same can be said about various action languages (Giunchiglia et al. 2004) as they are all based on McCain and Turner's causal logic.

Denecker *et al.* (1998) proposed a approach based on inductive definitions. In their formalism, the causal rule in the suitcase example is represented as

$$\begin{aligned}
&caus(open) \leftarrow init(up_1) \wedge holds(up_2) \wedge \neg init(\neg up_2),\\
&caus(open) \leftarrow init(up_2) \wedge holds(up_1) \wedge \neg init(\neg up_1),\\
&caus(open) \leftarrow init(up_1) \wedge init(up_2),
\end{aligned}$$

and for the two gears example, the following rules:

$$\begin{aligned}
&caus(gw(1)) \leftarrow caus(gw(2)),\\
&caus(gw(2)) \leftarrow caus(gw(1)),\\
&caus(\neg gw(1)) \leftarrow caus(\neg gw(2)),\\
&caus(\neg gw(2)) \leftarrow caus(\neg gw(1)).
\end{aligned}$$

Again we see that in this formalism, cyclic and acyclic causal rules need to be represented differently.

More recently, Strass and Thielscher (2010) considered a restricted causal language in the style of McCain and Turner's causal logic, but provided a different semantics in the style of Clark's completion (1978) and loop formulas (Lin and Zhao 2004). The causal rules from the two gears example are written as

$$\begin{aligned}
&\top : gw(1) \Rrightarrow gw(2),\\
&\top : gw(2) \Rrightarrow gw(1),\\
&\top : \neg gw(1) \Rrightarrow \neg gw(2),\\
&\top : \neg gw(2) \Rrightarrow \neg gw(1).
\end{aligned}$$

In general, a causal rule in this formalism is of the form

$$\Phi : l_1 \Rrightarrow l_2,$$

where Φ can be an arbitrary fluent formula, but l_1 and l_2 must be fluent literals. As mentioned, the semantics of these rules are defined using Clark completion and loop formulas. It is interesting to note that Lee (2004) did something similar by providing a translation from a subset of McCain and Turner's causal logic to logic program. When Lee's translation is applied to (12), it yields a set of rules very similar to the rules above.

In comparison, our proposal here allows for more general form of causal rules, and uses minimization instead of Clark's completion and loop formulas.

Concluding Remarks and Future Work

We have proposed to add a second minimization to the circumscriptive action theories in (Lin 1995). Intuitively, the original minimization in (Lin 1995) yields a closed-form solution for $Caused$ in terms of $Holds$. However when $Holds(f, do(a, s))$ is replaced by its pseudo-successor state axioms, the closed-form solution for $Caused$ may have some cycles which will then be eliminated by the proposed new minimization.

We have shown that our method is stronger than the original method in (Lin 1995) so that if the method in (Lin 1995) produces a set of successor state axioms, so will our new approach.

The main advantage of our method as compared to others that can handle cyclic causal rules is that we have used a uniform representation for both acyclic causal rules such as those in the suitcase example and cyclic ones such as those in the two gears example.

We plan to consider the following future work:

1. Show that when a set of causal rules of the form (11) has cycles, then the result of two minimizations can be captured by loop formulas as done in (Lee 2004) and (Strass and Thielscher 2010).
2. While for the two gears example, the various different approaches outlined above all yield the same results, it is worthwhile proving a result that formally relate, e.g. causal theories here and causal theories by McCain and Turner.
3. Implement a system similar to (Lin 2003) that can compile a causal theory into STRIPS-like systems and successor state axioms.
4. Define *actual cause* within our framework to capture properly concepts of causation and demonstrate that it conforms to intuition on examples from (Hopkins and Pearl 2007; Pearl 2009; Halpern and Pearl 2005).

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